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TECHNICAL NOTE

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A METHOD OF CALCULATING THE COMPRESSIVE STRENGTH OF Z-STIFFENED PANELS THAT DEVELOP LOCAL INSTABILITY

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SUMMARY

A method, based on the elastic theory for plate buckling and test results for 24S-T aluminum-alloy Z-stiffened panels, is shown for calculating the compressive strength of Z-stiffened panels that develop local instability. This method can be used to calculate the critical compressive stress above, as well as within, the elastic range. For stresses above three-fourths the compressive yield stress the method can be used for the approximate determination of the average compressive stress at maximum load.

INTRODUCTION

For maximum aerodynamic efficiency of a wing the compression panels in the wing should not buckle even when the applied load reaches high values. This requirement makes it necessary for aircraft designers to know the critical compressive stress - the stress at which buckling of the plate elements occurs - for the stiffened panels that make up the wing surfaces. In addition, the designer should have information available on the average compressive stress at maximum load, because the design of stressed-skin structures is based upon the strength of the section.

As an approach to solving these problems for Z-stiffened panels, a method of calculating the elastic theoretical buckling stress for local instability of Z-stiffened panels is presented. This method is extended to the calculation of the critical compressive stress above the elastic range on the basis of test results for 24S-T aluminum-alloy Z-stiffened panels. The same method may be used to calculate a reasonable value of average stress at maximum load for stresses above three-fourths the yield stress.

SYMBOLS

b	width of plate element of panel
t	thickness of plate element of panel
E	Young's modulus of elasticity (taken as 10.7×10^6 psi)
E_{sec}	secant modulus (ratio of ordinate to abcissa of stress-strain curve)
μ	Poisson's ratio (taken as 0.3)
σ_{cr}	critical compressive stress
$\bar{\sigma}_{max}$	average compressive stress at maximum load
σ_{cy}	compressive yield stress
ϵ_{cr}	calculated elastic critical compressive strain
k	nondimensional buckling stress coefficient dependent upon relative dimensions of cross section.

Subscripts:

F	flange
S	skin
W	web

CALCULATION OF THEORETICAL ELASTIC CRITICAL COMPRESSIVE STRESS

The theoretical elastic critical compressive stress σ_{cr} for plates may be calculated from the plate buckling equation

$$\sigma_{cr} = \frac{k\pi^2 E t^2}{12(1 - \mu^2)b^2} \qquad (1)$$

In this formula t and b are the thickness and the width, respectively, of the plate under consideration; E is Young's modulus; μ is Poisson's ratio; and k is a coefficient that is determined from the relative proportions of the cross section. The same equation may be adapted to plate assemblies. As applied to Z-stiffened panels it becomes

$$\sigma_{cr} = \frac{k_S \pi^2 E t_S^2}{12(1 - \mu^2) b_S^2} \tag{2}$$

where the subscript S refers to the skin or the plate between the stiffeners.

With equation (2) the elastic value of σ_{cr} can be calculated for Z-stiffened panels once the value of k_S is known. In order to evaluate k_S for Z-stiffened panels the actual panel cross section shown in figure 1(a) was idealized to that shown in figure 1(b). The values of k_S for this idealized panel are given by the curves of figure 2. These curves are similar to those for the web- and T-stiffened panels of reference 1 and were prepared in the same manner. Table 1 gives the computed values of k_S used for constructing these charts.

The basic assumption for calculating k_S is that, upon buckling, only rotation of the joints occurs (as in fig. 3(a)). It is recognized that, as the various dimensional ratios vary from one extreme to the other, this assumption is not completely true. As the ratio b_F/b_W decreases to 0.3 and less, the ability of the stiffener to resist lateral deflection of the web-flange joint (buckling as in fig. 3(b)) decreases. This deflection, which results in rotation of the stiffener, becomes more pronounced for wide stiffener spacing $\left(\text{small ratios of } b_W/b_S\right)$ because in such cases the skin is primarily responsible for the instability and therefore establishes a buckle pattern with a longer wave length than the wave length which results when some part of the stiffener is primarily responsible for the instability. Because the lateral deflection must be resisted in large measure by the bending stiffness of the flange as a beam of depth b_F, the larger the wave length the less the resistance to this deflection. It is believed, however, that the values of b_F/b_W and b_W/b_S in figure 2 are sufficiently large to prevent this lateral deflection (or stiffener rotation) from becoming great enough to reduce noticeably the values of k_S.

RESULTS AND DISCUSSION

Available test data on 24S-T aluminum-alloy Z-stiffened panels having formed stiffeners (see table 2) are presented to verify the calculated elastic theoretical stress given by equation (2). These test data include data obtained from references 2 and 3, along with some additional data not previously published. All of these experimental values of σ_{cr} were established by the strain-reversal method described in reference 4 and are for panels that are considered strongly riveted. The rivet pitch and diameter of these test panels is included in table 2. The effect of rivet pitch and diameter on the strength of panels is discussed in reference 5.

As in the case of reference 6, the test results are plotted against the calculated elastic critical compressive strain ϵ_{cr} for purposes of comparison with a compressive stress-strain curve. The equation for ϵ_{cr} is obtained by dividing both sides of equation (2) by E, giving

$$\epsilon_{cr} = \frac{k_S \pi^2 t_S^2}{12(1 - \mu^2) b_S^2} \qquad (3)$$

Figure 4 shows the data plotted in this manner. The compressive stress-strain curves shown represent the maximum and minimum compressive properties for the 24S-T aluminum-alloy sheet used in forming the test panels which buckled above the elastic range. The average yield stress for the material was 43.0 ksi.

The buckling data follow the stress-strain curves quite closely over the entire range. This fact indicates the possibility of using the secant modulus E_{sec} instead of Young's modulus E in equation (2), as was suggested in reference 6 for H-, Z-, and C-section plate assemblies. The use of the secant modulus for the buckling of plates has also been proposed by Gorard in reference 7. The test data indicate that the use of E_{sec} in equation (2) will give values that are slightly unconservative above the elastic range. When the average stress-strain curve was used, the amount of unconservatism was found to vary from 0.2 percent to 10.5 percent and to average about 6 percent. An attempt to reduce the scatter by referring each test point to its individual stress-strain curve showed only a little improvement. This result was probably due largely to the difficulty in obtaining a good representative stress-strain curve for each specimen because of the variation in properties caused by forming and by the use of different sheet material in the skin and stiffeners.

When buckling occurred at high stresses, values of the average stress at maximum load $\bar{\sigma}_{max}$ were found to be only slightly greater than σ_{cr}. These $\bar{\sigma}_{max}$ values have been included in figure 4 to show the close agreement between σ_{cr} and $\bar{\sigma}_{max}$ at high stresses. This result is similar to that for the H-, Z-, and C-sections in reference 6 for stresses greater than three-fourths the compressive yield stress σ_{cy}. Unlike the results obtained for these sections, the value of $\bar{\sigma}_{max}$ for the panels with stresses less than about $\frac{3}{4}\sigma_{cy}$ did not fall in a single curve and are not shown in figure 4 because of the considerable scatter. Because values of $\bar{\sigma}_{max}$ follow the stress-strain curves very closely for stresses above $\frac{3}{4}\sigma_{cy}$, the plate buckling equation with E_{sec} substituted for E can also be used to calculate approximate values of $\bar{\sigma}_{max}$ in this region. (This method has been suggested previously for H-, Z-, and C-section plate assemblies in reference 6.)

CONCLUSIONS

The following conclusions are based upon the local instability test results for 24S-T aluminum-alloy Z-stiffened panels having formed stiffeners strongly riveted to the skin:

1. The critical compressive stress for local instability may be calculated from the plate buckling equation with the secant modulus substituted for Young's modulus. This formula is

$$\sigma_{cr} = \frac{k_S \pi^2 E_{sec} {t_S}^2}{12(1 - \mu^2){b_S}^2}$$

where k_S is the buckling stress coefficient; t_S and b_S are the thickness and width of the skin plate, respectively; μ is

Poisson's ratio; and E_{sec} is the secant modulus. The results obtained by the use of this formula are reasonably accurate within the elastic range but tend to be on the average about 6 percent unconservative above the elastic range.

2. The values of the average stress at maximum load above three-fourths the yield stress are just slightly greater than the buckling stresses. An approximate value of the average compressive stress at maximum load, consequently, can also be calculated from the previous formula when this stress is above three-fourths the yield stress.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., Aug. 5, 1947

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TABLE 1

CALCULATED VALUES OF k_S

$\frac{b_W}{b_S}$	k_S					
	$\frac{b_F}{b_W}$			$\frac{b_F}{b_W}$		
	0.3	0.4	0.5	0.3	0.4	0.5
	$\frac{t_W}{t_S} = 0.50$			$\frac{t_W}{t_S} = 0.79$		
0.30	4.242	4.242	4.238	—	—	4.779
.40	4.172	4.170	4.157	—	—	4.648
.45	—	—	4.093	—	—	—
.50	4.095	4.061	3.422	—	—	4.533
.55	—	3.817	—	—	—	—
.60	3.787	3.208	2.377	4.435	4.420	4.374
.65	—	—	—	—	—	4.229
.70	2.785	2.357	1.747	4.298	4.258	4.012
.80	2.117	1.813	1.338	4.061	3.905	3.229
.90	1.683	1.431	1.057	3.639	3.316	2.565
1.00	1.364	1.158	.857	3.094	2.743	2.095
	$\frac{t_W}{t_S} = 0.63$			$\frac{t_W}{t_S} = 1.00$		
0.30	—	—	4.473	5.236	5.244	5.264
.40	—	—	4.346	5.075	5.090	5.098
.45	—	4.305	—	—	—	—
.50	4.267	4.256	4.230	4.989	4.980	4.970
.55	—	—	4.168	—	—	—
.60	4.154	4.124	3.709	4.863	4.871	4.855
.70	3.915	3.631	2.745	4.765	4.751	4.723
.80	3.259	2.812	2.104	4.626	4.594	4.460
.90	2.595	2.232	1.666	4.416	4.340	3.910
1.00	2.111	1.807	1.352	4.106	3.925	3.251

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TABLE 2

TEST DATA FOR 24S-T ALUMINUM-ALLOY Z-STIFFENED PANELS

THAT FAILED BY LOCAL INSTABILITY

Panel	$\frac{b_F}{b_W}$ (a)	$\frac{b_W}{b_S}$	k_S (fig. 2)	$\frac{b_S}{t_S}$	$\left(\frac{b_S}{t_S}\right)^2$	ϵ_{cr} (b)	σ_{cr} (ksi)	$\bar{\sigma}_{max}$ (ksi)	Rivet pitch (in.)	Rivet diameter (in.)
[a] $\frac{t_W}{t_S} = 0.51$										
[c]1	0.4	0.398	4.17	25.61	656	0.00575	40.0	41.2	7/8	3/16
[c]2	.4	.395	4.17	25.07	629	.00599	40.0	42.2	7/8	1/4
3	.4	.588	3.30	35.20	1239	.00241	23.5	26.5	$1\frac{1}{4}$	3/16
4	.4	.824	1.70	25.07	629	.00244	25.0	29.3	$1\frac{1}{4}$	3/16
5	.5	.300	4.25	34.91	1219	.00315	30.4	31.4	$1\frac{1}{4}$	3/16
6	.5	.364	4.17	34.24	1172	.00322	29.6	32.3	$1\frac{1}{4}$	3/16
[a] $\frac{t_W}{t_S} = 0.63$										
7	0.3	0.354	4.39	35.43	1255	0.00316	29.4	32.0	$1\frac{1}{4}$	3/16
8	.3	.450	4.30	35.52	1262	.00308	28.6	33.1	$1\frac{1}{4}$	3/16
9	.3	.375	4.37	50.06	2506	.00158	16.4	26.1	$1\frac{1}{4}$	3/16
[c]10	.4	.412	4.33	30.09	905	.00432	36.1	37.8	9/32	3/32
[c]11	.4	.419	4.32	30.25	915	.00427	35.8	38.0	9/16	1/8
[c]12	.4	.406	4.35	30.29	917	.00429	36.9	39.6	3/4	1/4
[c]13	.4	.415	4.33	30.33	920	.00425	36.8	38.7	7/8	1/4
[c]14	.4	.414	4.33	30.39	924	.00424	36.5	38.6	$1\frac{7}{32}$	1/4
[c]15	.4	.358	4.39	35.99	1295	.00306	30.7	35.0	7/8	1/8
[c]16	.4	.357	4.39	35.89	1288	.00308	30.1	33.9	$1\frac{7}{32}$	1/8
[c]17	.4	.357	4.39	35.59	1267	.00313	32.3	36.3	3/4	1/4
[c]18	.4	.357	4.39	35.84	1285	.00309	32.1	36.9	7/8	1/4
[c]19	.4	.356	4.39	36.03	1298	.00306	31.1	35.0	$1\frac{7}{32}$	1/4
[c]20	.4	.313	4.44	39.38	1551	.00259	27.5	33.2	9/16	3/32
[c]21	.4	.312	4.44	41.34	1709	.00235	25.9	33.5	$1\frac{7}{32}$	1/4
[c]22	.4	.485	4.28	25.05	628	.00616	42.2	43.4	7/8	1/4
23	.4	.314	4.44	50.43	2543	.00158	15.5	28.0	$1\frac{1}{4}$	3/16
24	.4	.381	4.37	50.13	2513	.00157	16.3	25.8	$1\frac{1}{4}$	3/16
25	.4	.379	4.37	47.14	2222	.00178	17.8	26.9	$1\frac{1}{4}$	3/16
26	.4	.747	3.20	25.63	657	.00440	35.3	36.3	$1\frac{1}{4}$	3/16
27	.4	.725	3.40	35.08	1231	.00250	24.5	27.8	$1\frac{1}{4}$	3/16
28	.4	.711	3.52	36.81	1355	.00235	24.1	24.9	$1\frac{1}{4}$	3/16
29	.4	1.002	1.78	26.05	679	.00237	24.4	29.4	$1\frac{1}{4}$	3/16
30	.4	.409	4.35	78.69	6192	.00063	7.1	17.7	$1\frac{1}{4}$	3/16
31	.5	.359	4.40	36.13	1305	.00305	29.7	33.5	$1\frac{1}{4}$	3/16
32	.5	.447	4.30	36.75	1351	.00288	29.7	32.5	$1\frac{1}{4}$	3/16

[a] Nominal ratio (nominal t_W = 0.064 in. for all panels).

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[b] $\epsilon_{cr} = \dfrac{k_S \pi^2 t_S^2}{12(1 - \mu^2) b_S^2}$, where $\mu = 0.3$.

[c] Unpublished data.

TABLE 2 — Concluded

TEST DATA — Concluded

Panel	$\frac{b_F}{b_W}$ (a)	$\frac{b_W}{b_S}$	k_S (fig. 2)	$\frac{b_S}{t_S}$	$\left(\frac{b_S}{t_S}\right)^2$	ϵ_{cr} (b)	σ_{cr} (ksi)	$\bar{\sigma}_{max}$ (ksi)	Rivet pitch (in.)	Rivet diameter (in.)
[a] $\frac{t_W}{t_S} = 0.79$										
33	0.3	0.474	4.55	49.26	2427	0.00169	19.1	27.8	1	5/32
34	.4	.358	4.70	54.97	3022	.00141	14.4	29.2	1	5/32
[c]35	.4	.638	4.37	24.80	615	.00642	39.1	39.7	7/8	3/16
36	.5	.318	4.76	48.55	2357	.00183	17.6	29.7	1	5/32
37	.5	.482	4.54	47.92	2296	.00179	17.9	26.9	1	5/32
38	.5	.486	4.53	46.18	2133	.00192	21.1	29.1	1	5/32
39	.5	.323	4.75	71.66	5135	.00084	8.1	22.5	1	5/32
[a] $\frac{t_W}{t_S} = 1.00$										
40	0.3	0.506	4.97	50.97	2598	0.00173	19.2	29.6	3/4	1/8
41	.3	.340	5.20	73.67	5427	.00086	9.2	24.2	3/4	1/8
[c]42	.4	.800	4.60	25.42	646	.00644	42.4	43.9	5/8	1/8
[c]43	.4	.753	4.68	26.92	725	.00583	42.7	43.9	5/8	5/32
44	.4	.506	4.97	52.26	2731	.00165	18.4	31.0	3/4	1/8
45	.4	.339	5.20	77.88	6065	.00078	7.5	25.8	3/4	1/8
46	.5	.569	4.90	35.20	1239	.00357	33.0	36.1	3/4	1/8
47	.5	.399	5.11	50.10	2510	.00184	20.0	32.0	3/4	1/8
48	.5	.720	4.67	35.14	1235	.00342	32.7	35.3	3/4	1/8
49	.5	.724	4.66	35.45	1257	.00335	31.6	33.1	3/4	1/8
50	.5	.506	4.98	49.75	2475	.00182	19.5	30.8	3/4	1/8
51	.5	.335	5.21	74.96	5619	.00084	8.6	25.5	3/4	1/8

[a] Nominal ratio (nominal $t_W = 0.064$ in. for all panels).

[b] $\epsilon_{cr} = \dfrac{k_S \pi^2 t_S^2}{12(1 - \mu^2) b_S^2}$, where $\mu = 0.3$.

[c] Unpublished data.

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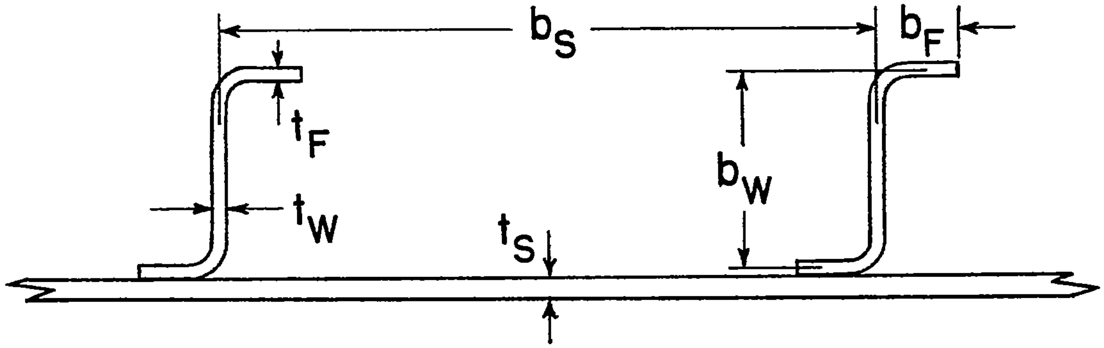
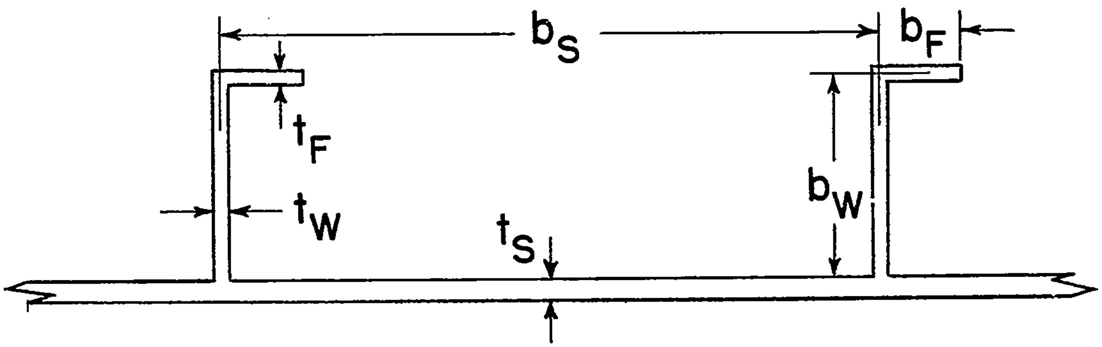


(a) Test panel.



(b) Idealized panel.

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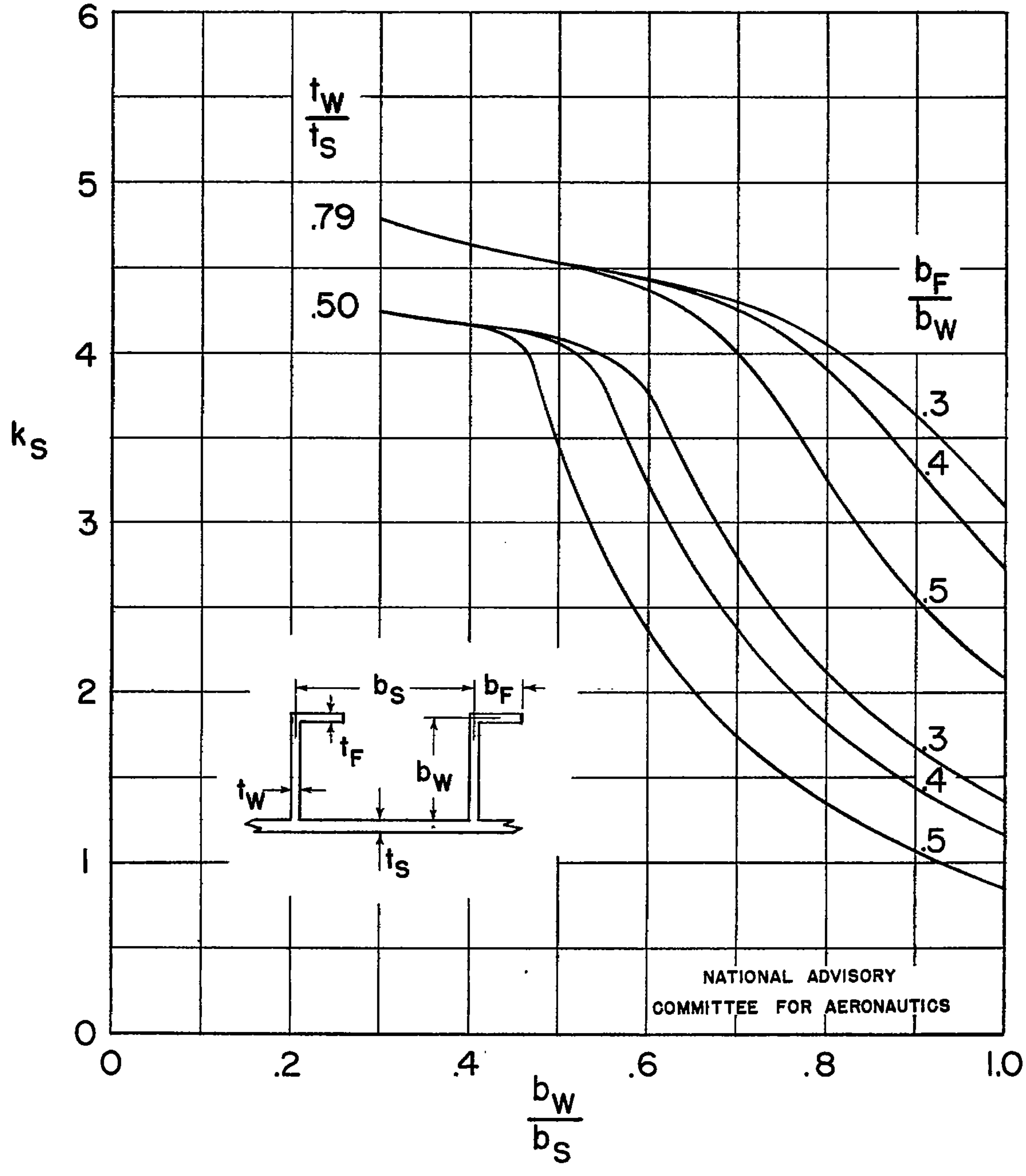
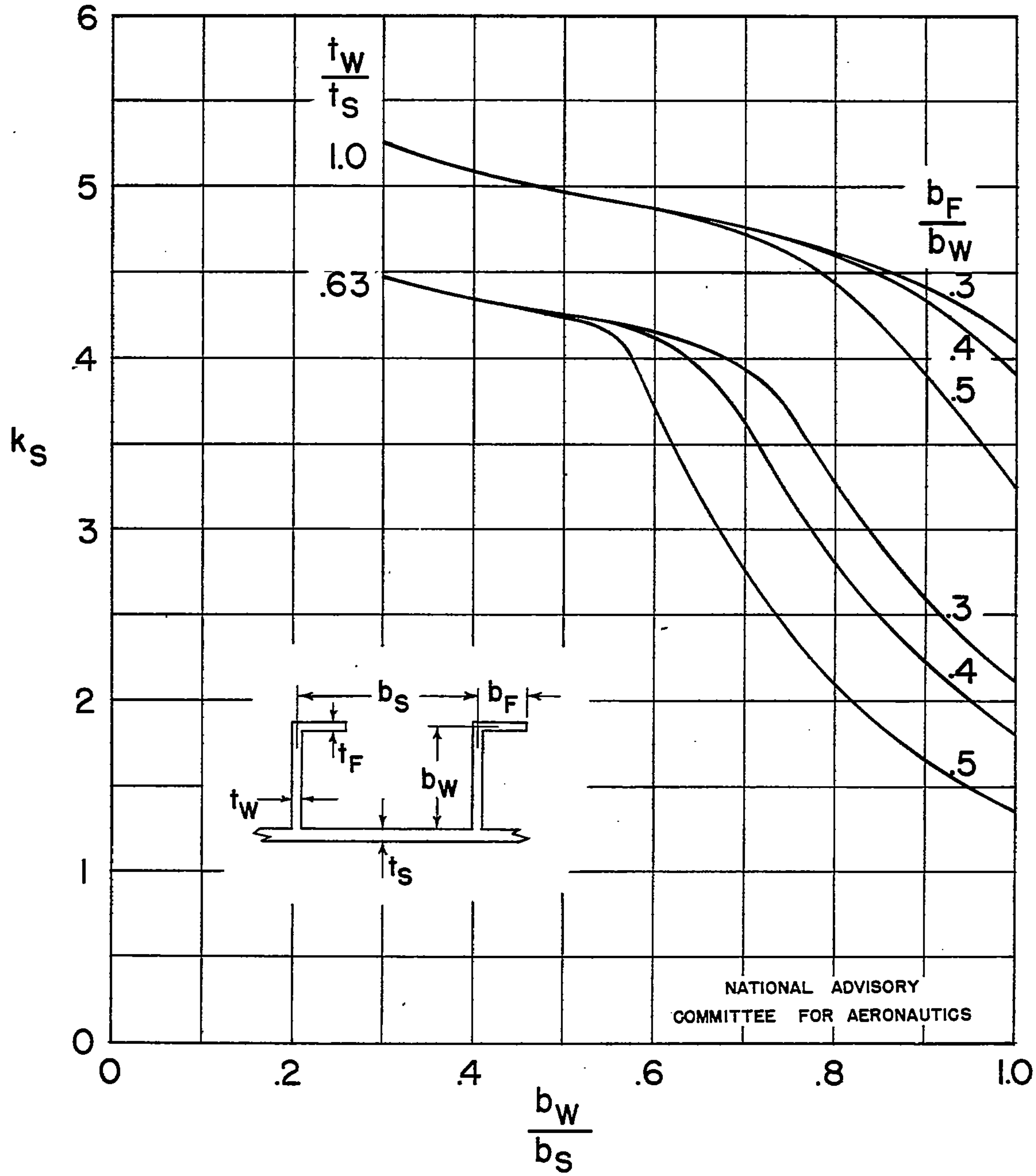
Figure 1.— Cross section and dimensions of Z-stiffened panels.



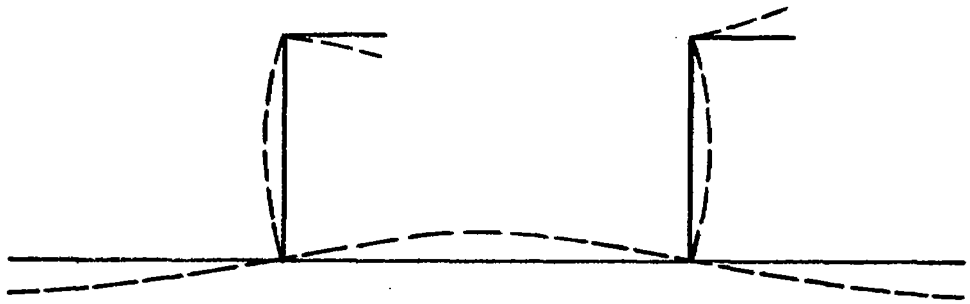
(a) $\frac{t_W}{t_S} = 0.50$ and 0.79.

Figure 2. — Values of k_S for a uniformly loaded idealized compression panel with Z-section stiffeners, $\frac{t_W}{t_F} = 1.0$.

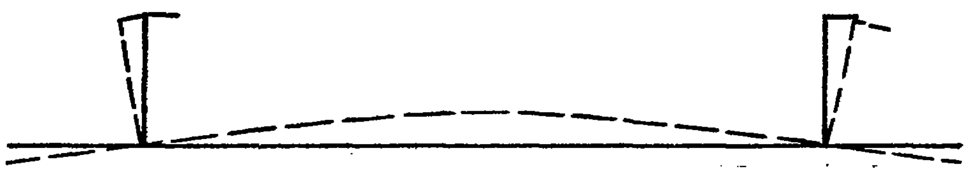


(b) $\frac{t_W}{t_S} = 0.63$ and 1.0.

Figure 2.— Concluded.



(a) Rotation of the joints.



(b) Rotation plus deflection of the web-flange joints.

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Figure 3.— Cases of local instability considered for a Z-stiffened panel.

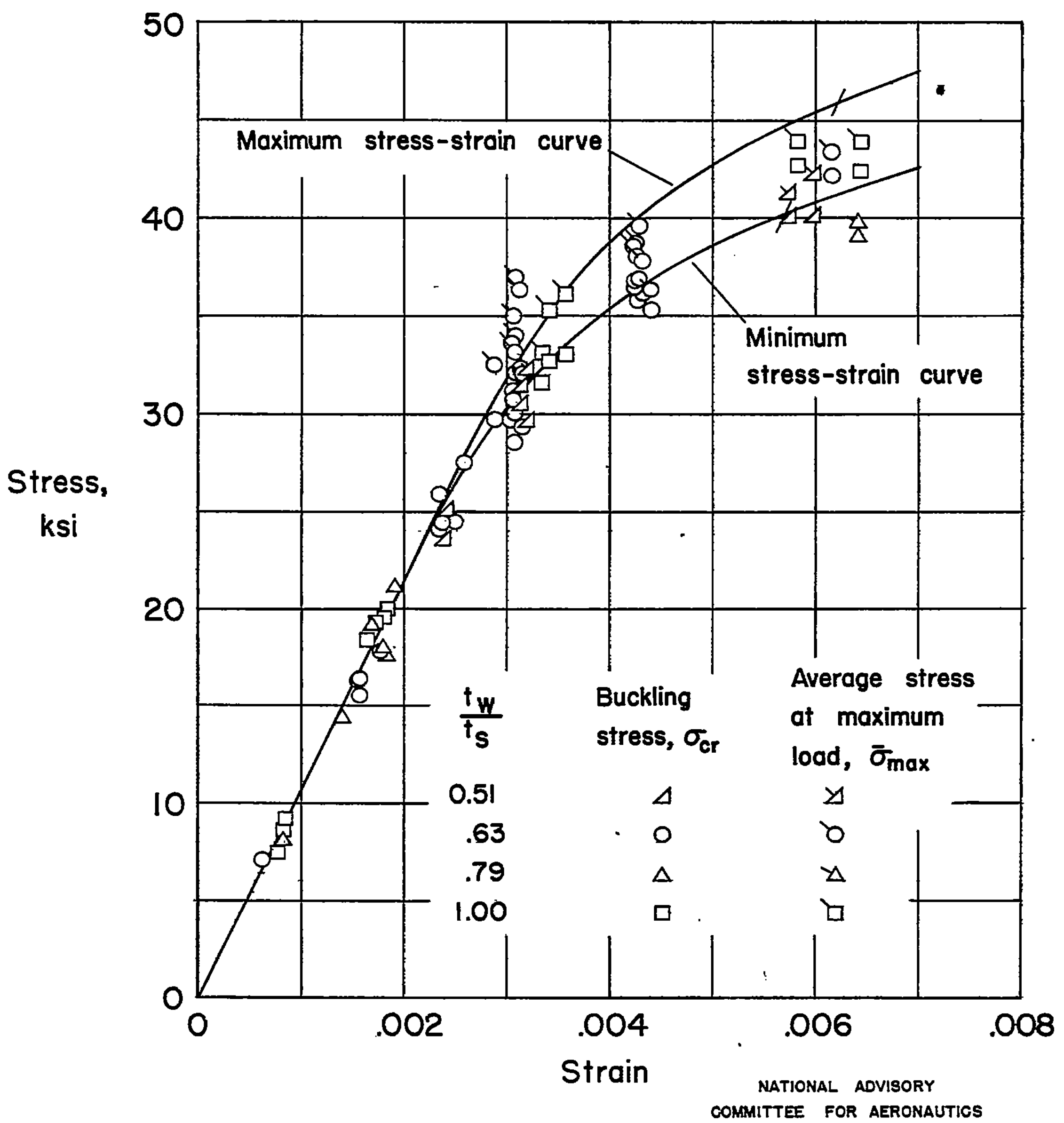

Figure 4.— Comparison of local instability test results with compressive stress-strain curves for 24S-T aluminum-alloy Z-stiffened panels. (Strain for Z-stiffened panels is calculated elastic critical compressive strain, ϵ_{cr}.)